WL03 DOUBLE-SLIT INTERFERENCE

SPH4U

CH 9 (KEY IDEAS)

- analyze and interpret the properties of two-dimensional mechanical waves in a ripple tank and relate them to light
- derive and apply equations involving the speed, wavelength, frequency, and refractive index of waves and apply them to the behaviour light
- analyze two-point-source interference patterns in a ripple tank and in the interference of light (Young's experiment) using diagrams
- derive and apply equations relating the properties of wave interference and wavelength
- outline the historical development of the particle and wave theories of light, including the development of new technologies and discoveries, and summarize the successes and failures of each theory
- apply the wave theory to the property of dispersion and determine the wavelengths of the colours of the visible spectrum

EQUATIONS

- Destructive Interference

$$\sin\theta_n = \frac{x_n}{L} = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}, n = 1, 2, 3, \ldots$$

- Constructive Interference

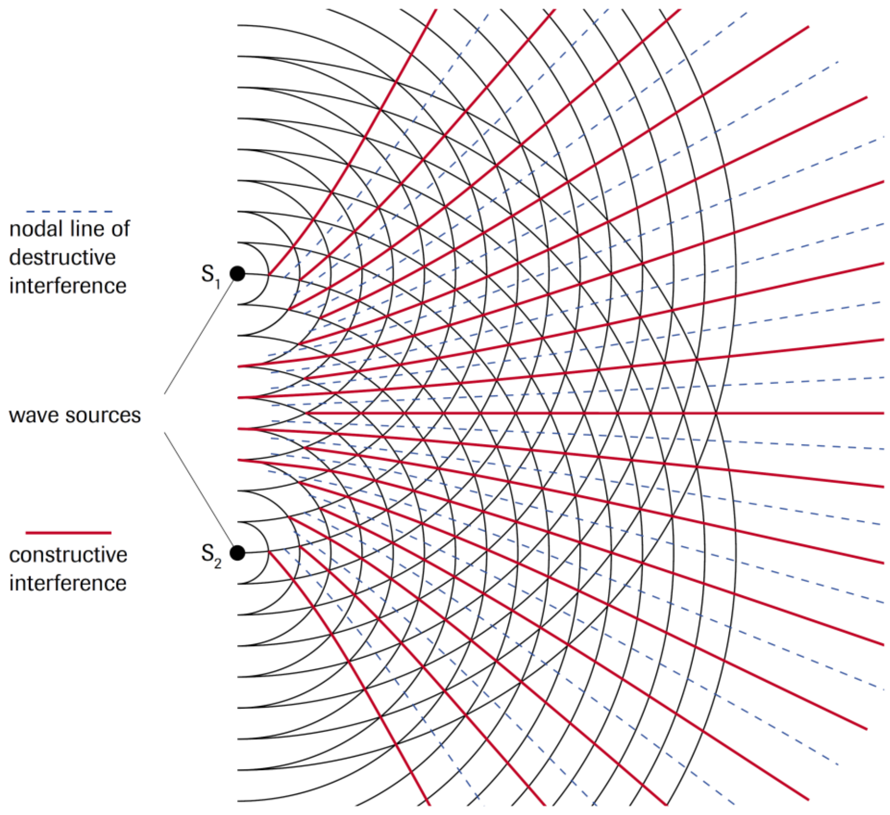
$$\sin\theta_m = \frac{x_m}{L} = \frac{m\lambda}{d}, m = 0, 1, 2, 3, \ldots$$

- Displacement between adjacent nodal lines

$$\frac{\Delta x}{L} = \frac{\lambda}{d}$$

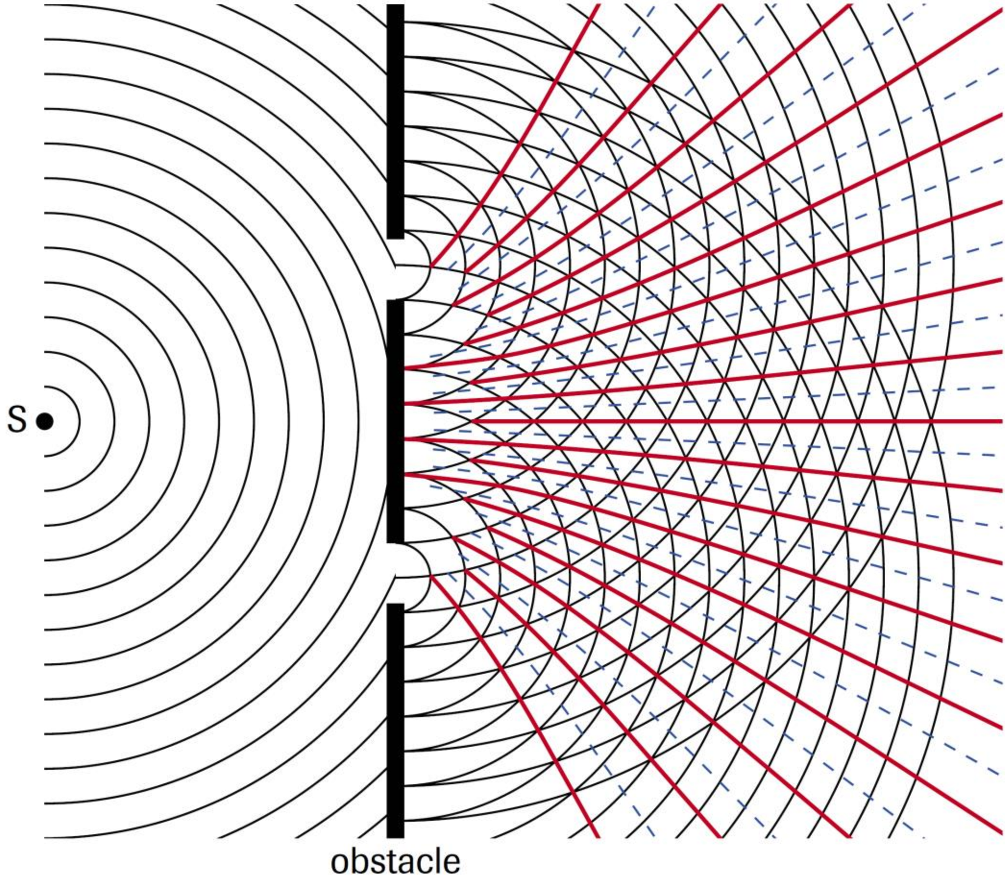
WAVE INTERFERENCE

- With water waves, it is easy to see the interference patterns made by two sources that are in phase
- With light, it was impossible to create the same patterns using two sources
    - The sources (incandescent lightbulbs) emitted light randomly, so the waves were always out of phase
    - The sources were too far apart compared to the extremely small wavelengths of light



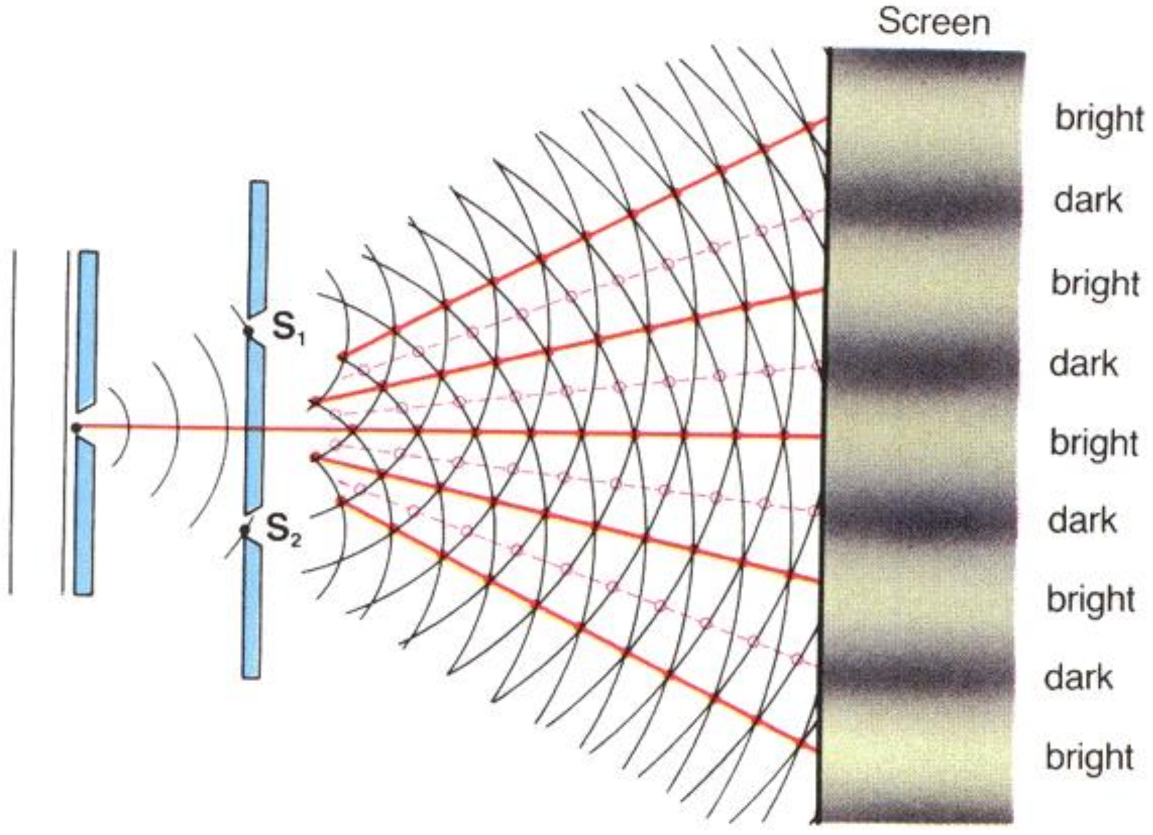
YOUNG'S DOUBLE-SLIT EXPERIMENT

- Thomas Young (1773-1829) used the Sun as the only source shone through two slits
- Since they were from the same source, the waves were in phase
- The distance between the slits was small enough to see interference patterns on a screen



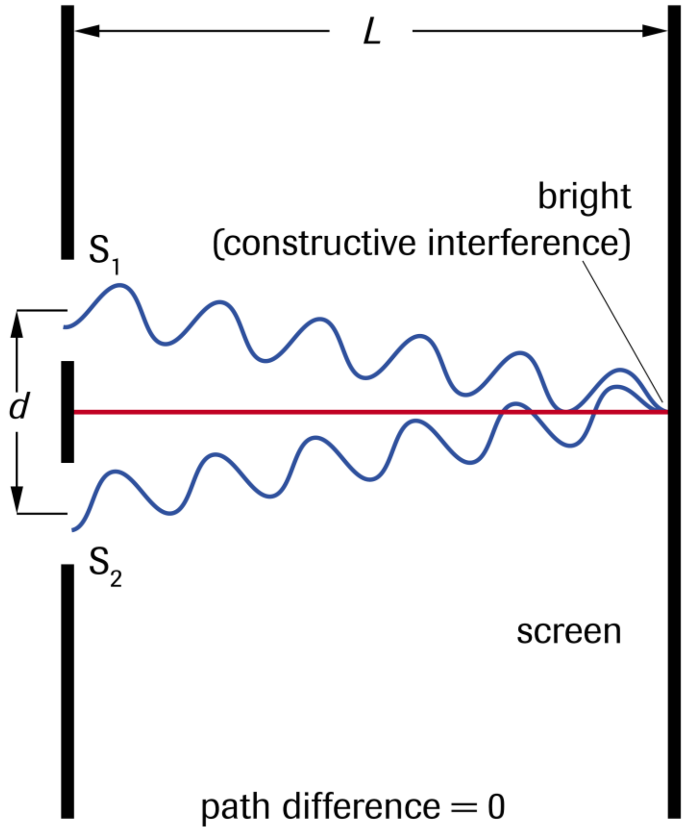
INTERFERENCE OF LIGHT WAVES

- **Interference Fringes:** the light (<u>maxima</u>) and dark (<u>minima</u>) bands produced by the interference of light



INTERFERENCE OF LIGHT WAVES – CONT.

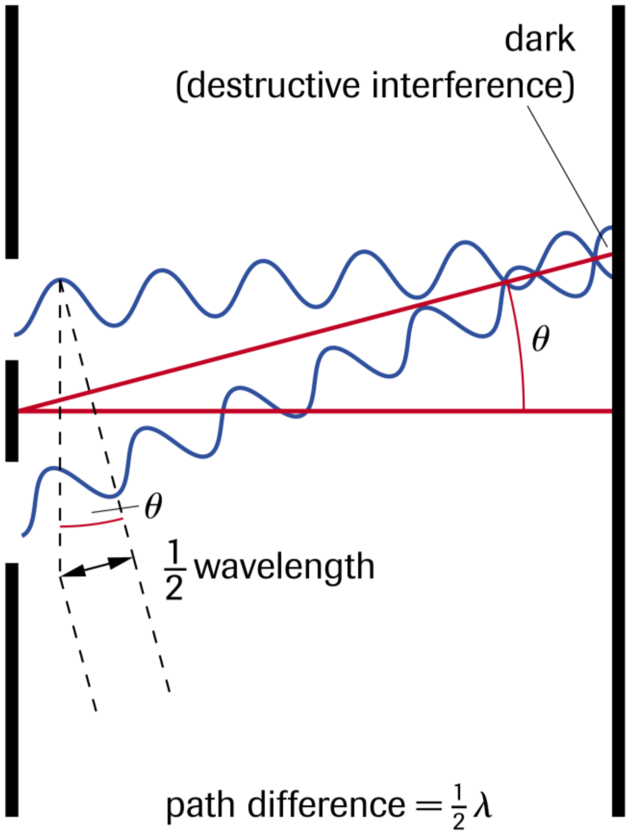
- As with two sources, the waves meeting along the perpendicular bisector are in phase
- This is the central, or zero-order maximum $(m = 0)$



INTERFERENCE OF LIGHT WAVES – CONT.

- When the path difference is $\frac{1}{2}\lambda$, the waves are now out of phase by 180°
- This is the first-order minimum $(n = 1)$
- With path difference $d \sin\theta_n$, destructive interference is found using

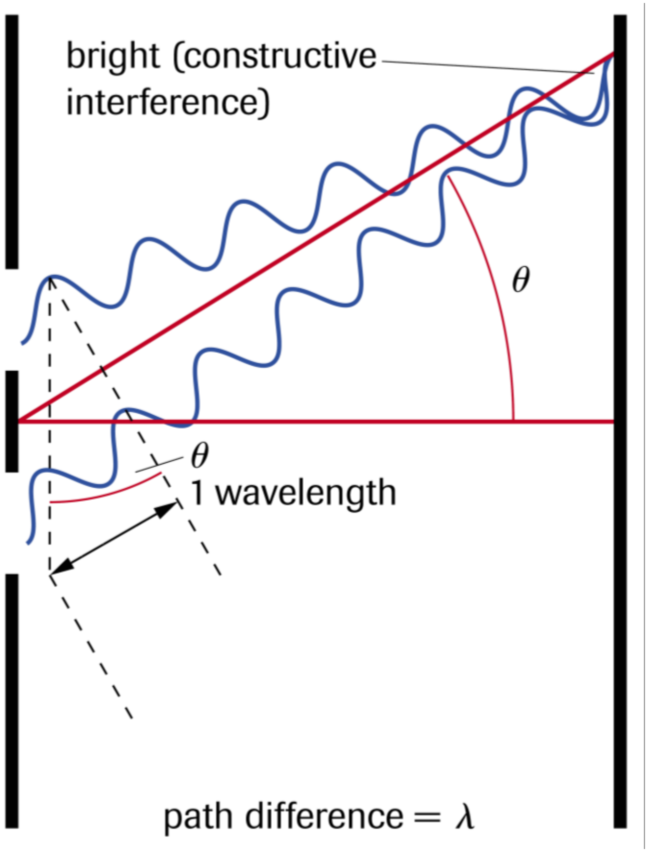
$$\sin\theta_n = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}, n = 1, 2, 3, \ldots$$



INTERFERENCE OF LIGHT WAVES – CONT.

- When the path difference is λ, the waves are back in phase
- This is our first-order maximum $(m = 1)$
- With path difference $d\sin\theta_n$, constructive interference is found using

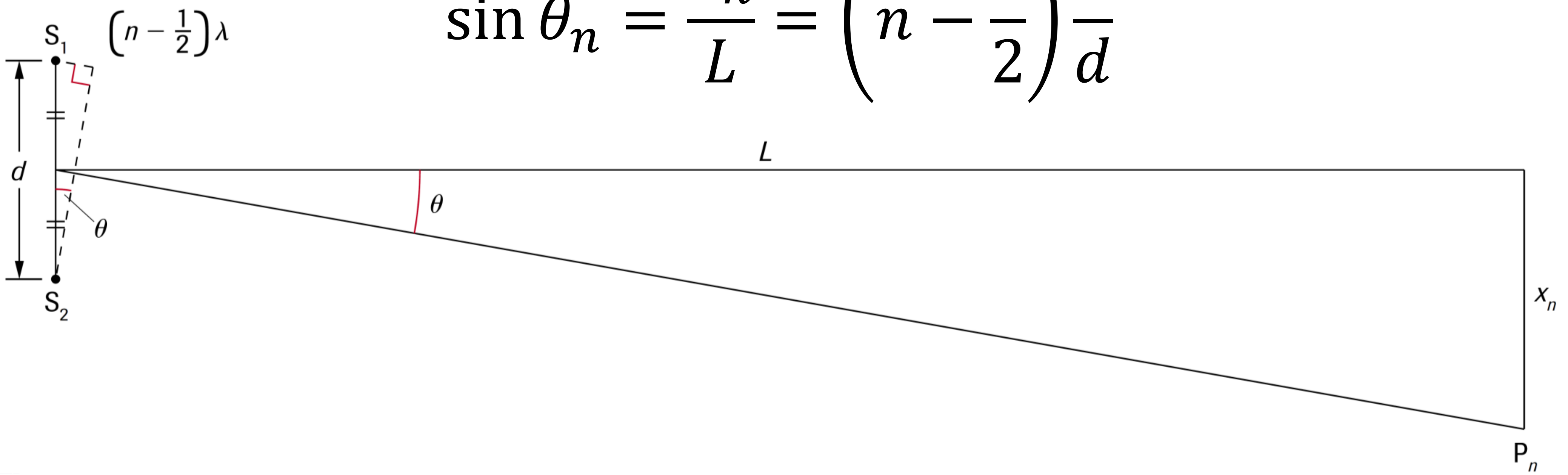
$$\sin\theta_m = \frac{m\lambda}{d}, m = 0, 1, 2, 3, \ldots$$



INTERFERENCE OF LIGHT WAVES – CONT.

- Since we can't measuring θ_n directly, we use $\sin\theta_n = \frac{x_n}{L}$ as before, so we get

$$\sin\theta_n = \frac{x_n}{L} = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$



INTERFERENCE OF LIGHT WAVES – CONT.

NOTES:

- $\sin\theta \approx \tan\theta$ for small values of θ
- The distance from the midpoint between the sources and the central maximum on the screen (L) is approximately equal to the distance from the midpoint to the first-order minimum, so we use the straight-across distance

INTERFERENCE OF LIGHT WAVES – CONT.

- For each nodal line, we can derive the respective x_n value

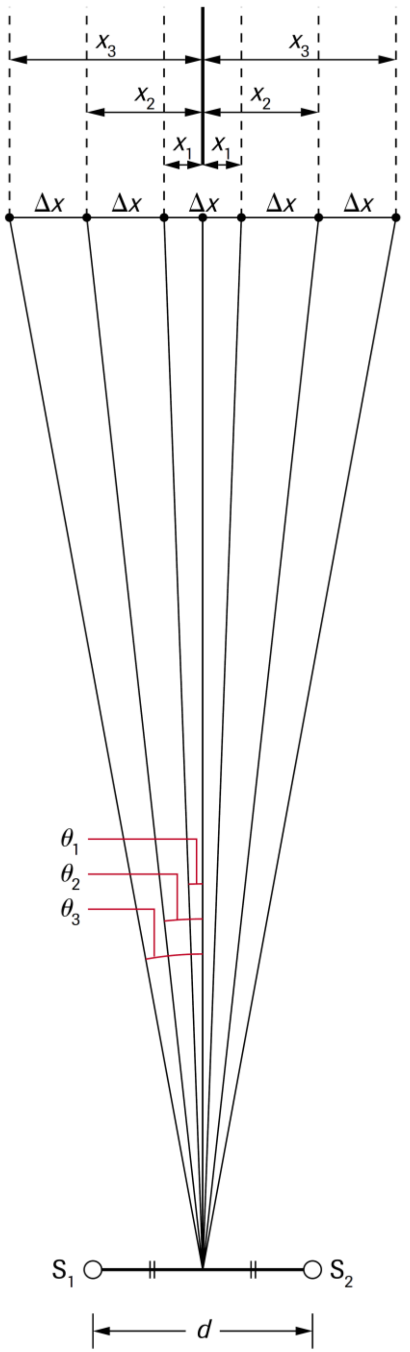
$$x_n = \left(n - \frac{1}{2}\right)\frac{L\lambda}{d}$$

$$x_1 = \left(1 - \frac{1}{2}\right)\frac{L\lambda}{d} = \frac{L\lambda}{2d}$$

$$x_2 = \left(2 - \frac{1}{2}\right)\frac{L\lambda}{d} = \frac{3L\lambda}{2d}$$

$$x_3 = \left(3 - \frac{1}{2}\right)\frac{L\lambda}{d} = \frac{5L\lambda}{2d}$$

$$\vdots$$



INTERFERENCE OF LIGHT WAVES – CONT.

- Since our distance L is so large compared to d, the lengths to the different nodal lines are similar enough to consider L as constant
- By defining the distance between nodal lines as Δx, we can use any of the previous x_n equations to derive

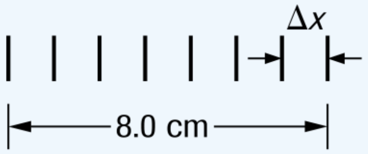
$$\frac{\Delta x}{L} = \frac{\lambda}{d}$$

  - Δx – distance between adjacent nodal lines [m]
  - L – distance from the slits to the screen [m]
  - λ – wavelength [m]
  - d – slit separation distance [m]

PROBLEM 1

You are measuring the wavelength of light from a certain single-colour source. You direct the light through two slits with a separation of 0.15 mm, and an interference pattern is created on a screen 3.0 m away. You find the distance between the first and the eighth consecutive dark lines to be 8.0 cm. At what wavelength is your source radiating?

PROBLEM 1 – SOLUTIONS



$L = 3.0 \text{ m}$

$\lambda = ?$

8 nodal lines $= 7\Delta x$

$$\Delta x = \frac{8.0 \text{ cm}}{7} = 1.14 \text{ cm} = 1.14 \times 10^{-2} \text{ m}$$

$d = 0.15 \text{ mm} = 1.5 \times 10^{-4} \text{ m}$

$$\frac{\Delta x}{L} = \frac{\lambda}{d}$$

$$\lambda = \frac{d\Delta x}{L}$$

$$= \frac{(1.14 \times 10^{-2} \text{ m})(1.5 \times 10^{-4} \text{ m})}{3.0 \text{ m}}$$

$$\lambda = 5.7 \times 10^{-7} \text{ m}$$

The wavelength of the source is 5.7×10^{-7} m, or 5.7×10^{2} nm.

PROBLEM 2

The third-order dark fringe of 652-nm light is observed at an angle of 15.0° when the light falls on two narrow slits. How far apart are the slits?

PROBLEM 2 – SOLUTIONS

$n = 3$ $\qquad \theta_3 = 15.0°$

$\lambda = 652\ \text{nm} = 6.52 \times 10^{-7}\ \text{m}$ $\qquad d = ?$

$$\sin\theta_n = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$

$$d = \frac{\left(n - \frac{1}{2}\right)\lambda}{\sin\theta_n}$$

$$= \frac{\left(3 - \frac{1}{2}\right)(6.52 \times 10^{-7}\ \text{m})}{\sin 15.0°}$$

$$d = 6.30 \times 10^{-6}\ \text{m}$$

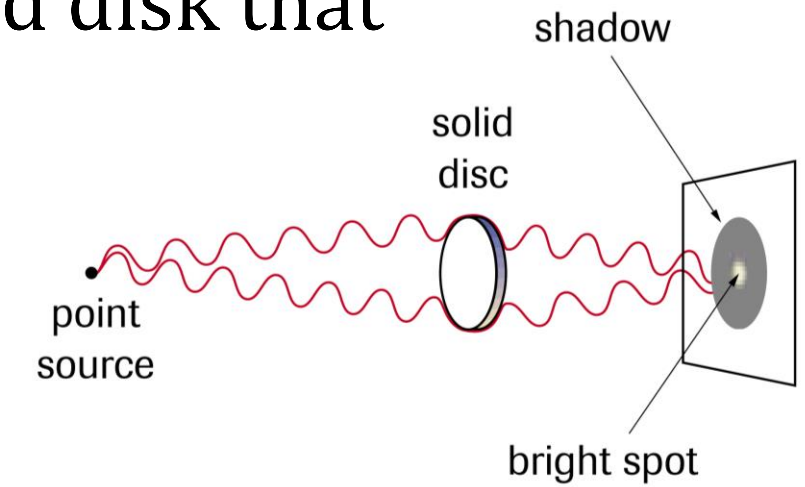
The slit separation is 6.30×10^{-6} m.

FURTHER DEVELOPMENTS IN THE WAVE THEORY OF LIGHT

- We use monochromatic light (usually lasers) to clearly see the interference patterns of light
- **Monochromatic:** composed of only one colour; possessing only one wavelength
- Young's arguments were not accepted until further mathematical confirmation by Augustin Fresnel (1788-1827)

POISSON'S BRIGHT SPOT

- Simon Poisson was a mathematician who strongly supported the particle theory of light
- He argued that if the wave theory were true there would be constructive interference around a solid disk that would appear as a bright spot
- This was later proven to be true experimentally by Dominique Arago in 1818 and was named after Poisson



SUMMARY: WAVE INTERFERENCE: YOUNG'S DOUBLE- SLIT EXPERIMENT

- Early attempts to demonstrate the interference of light were unsuccessful because the two sources were too far apart and out of phase, and the wavelength of light is very small.
- Thomas Young's crucial contribution consisted of using one source illuminating two closely spaced openings in an opaque screen, thus using diffraction to create two sources of light close together and in phase.
- In Young's experiment a series of light and dark bands, called interference fringes, was created on a screen, placed in the path of light, in much the same way as those created in the ripple tank.
- The relationships $\sin\theta_n = \frac{x_n}{L} = \frac{\left(n-\frac{1}{2}\right)\lambda}{d}$ and $\frac{\Delta x}{L} = \frac{\lambda}{d}$ permit unknowns to be calculated, given any three of λ, Δx, L, θ, d, and n.
- Young's experiment supported the wave theory of light, explaining all the properties of light except transmission through a vacuum.

PRACTICE

Readings

- Section 9.5 (pg 469)

Questions

- pg 475 #2,3,5,7,8